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# *Active Learning for Causal Bayesian Network Structure with Non-symmetrical Entropy*

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# T e c h n i c a l  R e p o r t

## Foreword

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# Active Learning for Causal Bayesian Network Structure with Non-symmetrical Entropy

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**Abstract.** Causal knowledge is crucial for facilitating comprehension, diagnosis, prediction, and control in automated reasoning. Active learning in Bayesian networks involves interventions by manipulating specific variables or their interactions, and observing the patterns of change over the other variables to derive causal relationships for knowledge discovery. In this paper, we propose a new active learning approach that supports interventions with node selection. Our method admits a node selection criterion based on non-symmetrical information entropy and a stop criterion based on minimizing structure entropy of the resulting networks. We examine the technical challenges and practical issues in developing effective node selection and stopping criteria in our method. Experimental results on a set of benchmark Bayesian networks are promising. The proposed method is applicable in many real-life applications where multiple instances are simultaneously sampled as a data set in each active learning step.

**Keywords:** Bayesian networks, active learning, intervention, non-symmetrical entropy, node selection, stop criterion

## 1 Introduction

Causal knowledge is important for comprehension, diagnosis, prediction and control in automated reasoning. Causal Bayesian networks are extensions to Bayesian networks that explicitly and concisely represent causal knowledge as variables and their directed graphical relationships in uncertain domains [11]. This research focuses on learning causal knowledge from data that corresponds to learning the structure of causal Bayesian networks for knowledge discovery. A major research challenge is to learn causal knowledge from both observational and interventional data. Observational data are derived from passive observations when the underlying system evolves autonomously. Interventional data are observed when some variables are actively manipulated to fixed values, while other variables evolve autonomously according to the underlying system mechanisms; such data directly reflect the effects of the active manipulation of certain variables on the rest of the system.

Most of the existing Bayesian network structure learning methods deal with observational data [1, 6]. Recently, some new learning methods have been proposed to combine observational data with interventional data [2, 10, 14]. Cooper and Yoo [2] identified the possible assumptions for probability updates with both observational and interventional data and proposed a method to update the probabilities with the combined data. Tong and Koller [14] developed an active learning method to guide the interventions to collect new interventional data for further structure probability updates. Eberhardt et al. [5] proved that, under ideal conditions with causal Markov assumption and faithfulness assumption (and ideal distributions), the number of interventions required to identify the causal relationships among $N$ variables is $N-1$ when only one variable can be manipulated each time, and the number of interventions is $\log_2 N$ when multiple variables can be manipulated simultaneously. Meganck et al. [9] assumed that the correct complete partial directed acyclic graphs (CPDAGs) can be learned from the observational data and the directions of the un-directed edges in the CPDAGs can be determined with interventional data.

Active learning in Bayesian networks involves interventions by manipulating specific variables or their interactions, and observing the patterns of change over the other variables to derive causal relationships for knowledge discovery. In previous active learning work [10, 14], the interventional data are assumed to include one instance at each active learning step. In this work, we consider a new scenario: a data set of multiple instances is collected when one variable is manipulated at each active learning step. Such experiments arise in many real-life applications. For instance, in measuring protein expression levels with flow cytometry in biology, the expression levels of some proteins (as variables) can be intervened and their effects on the expression levels of other proteins are observed from many cells at a time, and the observations of protein expression levels from one cell are the values in one instance [4, 12].

With an interventional data set, we can determine the causal influences of the manipulated variables on other variables based on the theory of *causality with agency*: manipulating causes can change the effects but not vice versa [15]. In practice, marginal distributions of the variables are used to detect causal influence. If the marginal distribution of variable $B$ changes when variable $A$ is manipulated to different values, we say that variable $A$ *precedes* variable $B$ in causal ordering, variable $A$ is a cause of variable $B$, and variable $B$ is an effect of variable $A$.

There are different definitions of intervention: perfect intervention, imperfect intervention [8, 13], and uncertain intervention [4]. Different types of intervention have different effects on the Bayesian network structure learned from data. We will focus on perfect intervention in this work. When we manipulate a variable under perfect intervention, the manipulated variable takes the value we specify in the intervention. This is what we mean by manipulation in the general sense.

Our objective is to learn the causal Bayesian network structure that achieves the specified structure accuracy with a minimal number of interventions, when the interventional data comprise of a data set at each active learning step. Specifically, we address the following questions in the active learning for causal Bayesian network structure: 1) What is a good criterion for selecting the nodes for new interventions, with respect to "correctness" in terms of entropy of the learned structure? 2) What is the effect of the stop criterion on the learned structure in the learning process? and 3)

What is the probability of a positive finding in the next immediate intervention, given the constraint that only one more intervention can be performed?

We introduce a new active learning algorithm for causal Bayesian networks with a node selection criterion based on a measure of non-symmetrical entropy and a learning stop criterion based on the structure entropy of the resulting Bayesian networks. The definition of the non-symmetrical entropy is motivated by the non-symmetrical nature of the interventions. We examine the effectiveness and efficiency of the proposed method on identifying causal relationships based on a set of benchmark Bayesian networks; we also compare the results with some other major methods involving node selection with symmetrical entropy, random node selection, and observational data only.

# 2 Method

## 2.1 Causal Bayesian networks

A causal Bayesian network [11] is a directed acyclic graph (DAG), in which each node corresponds to a distinct variable $X_i$ in the domain, and each edge corresponds to a causal influence from the parent variable to the child variable. The parent variable of an edge is the variable at the tail of the edge, and the child variable is the variable at the head of the edge. The meaning of "causal" in causal Bayesian networks is from the interpretation of the edges in the model: The causal influence from the parent variable to the child variable means that, when we manipulate the parent variable by fixing its state to a specific value, we can observe the change in the probability distribution of the child variable. If there is no causal influence from one variable A to another variable B, there will be no edge from variable A to variable B. Moreover, when one variable is manipulated, the causal influence relationship between other variables will not change, and the conditional probability of the child variable given its parents will be the same. Under the causal Markov assumption, each variable is independent of its ancestors given the values of its parents. The joint probabilities in the domain can be represented as

$$p(X_1,..., X_n) = \prod_i p(X_i \mid Pa(X_i))$$

where $Pa(X_i)$ denotes the parents of $X_i$ in the causal Bayesian network. In this paper, we will use "node" and "variable" interchangeably. A good definition of causal Bayesian network and its properties can be found in Pearl's book [11].

## 2.2 Active learning

Active learning is different from the ordinary passive learning. Passive learning works with a set of readily available data; the data set does not change in the learning process. In active learning, we can sample new data in the learning process. In the active learning of causal Bayesian networks [10, 14], the process starts with an

available data set, and the probabilities of the edges are estimated from the available data (all observational and interventional data). With the edge probabilities, a node is selected with a certain criterion for intervention, and a new instance is collected in order to maximally reduce the expected structure entropy. The process can be repeated until the goal is reached.

Estimating the edge probabilities is an important part of the active learning process. For every pair of variables, three possible situations between them are usually considered: an edge from $A$ to $B$ ($A \to B$), an edge from $B$ to $A$ ($A \leftarrow B$), or no edge between $A$ and $B$ ($A \perp B$). The probabilities of the edges given the available data $D$ and domain knowledge $K$ are defined as

$$\Pr(A \to B \mid D, K) = \sum_{A \to B \in E(G)} \Pr(G \mid D, K)$$

where $\Pr(G \mid D, K)$ is the probability of the Bayesian network $G$ given the data $D$ and domain knowledge $K$, and $E(G)$ is the set of edges in Bayesian network $G$. In the following discussions, $D$ and $K$ will be omitted for brevity. The probabilities of $A \leftarrow B$ and $A \perp B$ are similarly defined as the probability of $A \to B$. The edge entropy is defined as in [14]:

$$\begin{aligned} H_S(A,B) = &- p(A \to B) \log p(A \to B) \\ &- p(A \leftarrow B) \log p(A \leftarrow B) \\ &- p(A \perp B)) \log p(A \perp B) \end{aligned}$$

The structure entropy of Bayesian network $G$ is defined as

$$H_S(G) = \sum_{A,B} H_S(A,B)$$

In the previous work [14], the edge probabilities are estimated approximately with Markov Chain Monte Carlo (MCMC). In contrast, we estimate the edge probabilities with an exact method proposed by Koivisto [7], since the exact edge probabilities can provide more information for node selection. Koivisto utilized the intuition that the order of the parents of a variable is irrelevant to the variable's probability estimation, and applied forward and backward dynamic programming and fast truncated Mobius transform to estimate all the edge probabilities in $O(n2^n)$ time, where $n$ is the number of variables in the domain. When the interventional data is combined with observational data, the instances with the variable intervened will not be used in calculating the probability of the family with the intervened variable as the child (the assumptions and the method can be referred to Cooper and Yoo's work [2]). Koivisto's exact method can be applied to domains with a moderate number of variables (around 25). Our intention here is to closely examine the performance of an exact estimation method for the proposed node selection criterion.

### 2.3 Selecting nodes for new interventions

In the previous work [14], node selection for intervention is based on the expected posterior loss of the structure entropy. The expected posterior loss for all the possible node selection needs to be estimated, and subsequently one node is selected for intervention to collect a new interventional instance.

We consider the situation where a data set will be collected when one variable is under one intervention. The interventional data set will show whether the changed value of the manipulated variable will affect the probability distributions of the other variables. The change from the probability distributions of the other variables can provide causal information between the interventional variable and all the other variables, and can help reduce the uncertainty of the causal relationships between the interventional variable and all the other variables.

We choose the node with maximum node uncertainty for intervention because it is computationally not feasible to calculate the expected posterior loss of the multiple instances in the entire data set at each step. The node uncertainty between a variable and all the other variables can be estimated under two different conditions:

$$U_{NS}(A) = \sum_{B} \left( -\Pr(A \to B) * \log(\Pr(A \to B) - (1 - \Pr(A \to B)) * \log(1 - \Pr(A \to B)) \right) \quad (1)$$

$$U_{S}(A) = \sum_{B} H_{S}(A, B) \quad (2)$$

The first case $U_{NS}$ considers two conditions between variable $A$ and the other variables: the probabilities whether there is an edge from $A$ to other variables or not. The second case $U_{S}$ considers the three possible conditions between variable $A$ and the other variables: $A \to B$, $A \leftarrow B$, and $A \perp B$. The second case is generally used in Bayesian network structure leaning.

We refer to $U_{NS}$ as *non-symmetrical entropy* and $U_{S}$ as *symmetrical entropy*. The definition of the non-symmetrical entropy is motivated by the non-symmetrical nature of the intervention. In an intervention, we can manipulate only one variable in a pair of variables to derive the causal information between the pair: whether or not the manipulated variable affects the non-manipulated variable. We cannot derive causal information from the non-manipulated variable to the manipulated variable. If both variables are manipulated, we cannot derive useful causal information between this pair of variables from the interventional data.

Besides examining the effects on node selection with these two measures, we also consider random node selection for intervention and selection using observational data only (i.e., there is no interventional variable in new data collection at each step of the active learning process).

### 2.4 Stop criteria for causal structure learning

Another main problem in applying Bayesian network learning for causal knowledge discovery in practice is to decide when to stop the learning process– when do we think that the learned causal Bayesian network is good enough? The intuitive way is to choose a fixed number of interventions as the stop criterion. The disadvantage of this approach is that there is no guarantee on the quality of the learned Bayesian network structure. We propose to use certain “acceptable” entropy of the learned structure as the stop criterion. The ideal entropy of the learned structure is 0; however, it is difficult in practice to reach the ideal condition. We consider the effects of the

different values of entropy of the learned structure as the stop criteria on the accuracy of the learned structures.

## 3 Experiments

The proposed method has been tested in experiments with the same benchmark Bayesian networks as those reported in Tong and Koller's work [14]: Cancer network (as shown in Figure 1), Asia network, and Car network. There are 5 variables in Cancer network, 8 variables in Asia network and 12 variables in Car network respectively. We conducted the simulations under MATLAB[1] (version 7) with the support of the BDAGL package [3]. The machine used is a Dell OptiPlex GX280 desktop with 1 Gigabyte memory and 3GigaHz Intel processor.

The experiment setup is as follows:

1. Choose a Bayesian network from Cancer network, Asia network, or Car network as the ground truth Bayesian network;
2. Sample an observational data set with 200 instances from the ground truth Bayesian network;
3. Estimate the edge probabilities and structure entropy with the available data (and domain knowledge, if any);
4. Check the stop criterion. If the stop criterion is satisfied, stop the learning process; otherwise, continue;
5. Select one node for intervention based on the node uncertainty measures from non-symmetrical entropy, symmetrical entropy, random node selection for intervention, or without interventional node;
6. Generate a new interventional data with 200 instances from the ground truth Bayesian network with the selected interventional variables; return to step 3).

In the experiments, the edge probabilities are estimated with the exact method from Koivisto [7]. The uniform prior of Bayesian network structures is used. We tested two stop criteria in our experiments - the number of interventions or the structure entropy of the learned Bayesian networks. In the latter, the maximum number of interventions is set to 50. This is because we had observed that the structure entropy of the learned Bayesian network would not reach certain small values with symmetrical entropy, even if a very large data set is sampled. The size of the interventional data is 200 instances in each intervention, which is more realistic than an ideal distribution estimated, e.g., as discussed in Eberhardt et al. [5], for each intervention.

In the experiments, when one variable is selected for intervention, the links pointing to this variable will be removed from the graph and this variable will be set to a fixed value. The values of other variables are sampled based on the Bayesian network structure and the original conditional probabilities. In addition, one variable can be selected for more than one round of intervention in the active learning process, since the probabilities of the variables from finite data are not ideal.

We used the original conditional probabilities in the Bayesian networks first. To test whether the specific values of the conditional probabilities in the original

[1] http://www.mathworks.com/products/matlab/

Bayesian networks will affect the conclusions, we also conducted experiments with the same Bayesian network structures but randomized conditional probabilities. The conclusions from the experiments with the randomized conditional probabilities are similar to the results with the original conditional probabilities. The following sections will discuss the experimental results based on the Cancer network. The results are consistent over all the benchmark Bayesian networks tested.

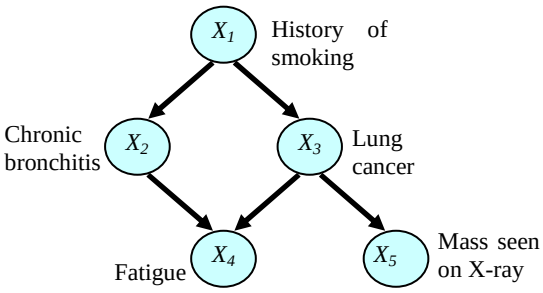


**Fig. 1.** Cancer Bayesian network

### 3.1 Number of interventions vs. structure entropy

In the first experiment, we tested the relationship between the number of interventions and the entropy of the learned structures. The objective is to show how the entropy of the learned structures varies with the different node selection methods, when the number of the interventions is the same. The maximum number of interventions is set to 6, because the structure entropy of the learned Bayesian networks with more than 6 interventions are observed to be very low. The programs ran 8 hours and finished 608 repeated experiments[2] on the Cancer network (about 48 seconds for one experiment). The results are shown in Figure 2.

In Figure 2, the lines represent the change of the average structure entropy with the number of interventions. Figure 2 shows that, with the same number of interventions, node selection with non-symmetrical entropy can derive a Bayesian network with the lowest entropy ( also with the smallest variance on average), which means the structure of the learned Bayesian network is more certain. The highest structure entropy is derived from observational data when the same number of data items is collected as that of the interventional data at each active learning step.

The entropy of Bayesian network structure learned with the random node selection and node selection with the symmetrical entropy fall between those of the node selection with non-symmetrical entropy and the observational data. This is consistent with our expectation, since the intervention is non-symmetrical in nature and the interventional data can provide more causal information about the probabilities between the manipulated variable and other variables. If there is a real edge from the manipulated variable to one other variable, the probability of this edge should

[2] We distinguish between the terms “intervention” and “experiment” here. “Intervention” means to manipulate the variables and observe other variables. “Experiment” means to run the method for testing. In Figure 2 and similarly for other figures, “6” is the maximum number of interventions.

increase with the interventional data, and the non-symmetrical entropy will decrease. However, the symmetrical entropy may not decrease since we do not have idea about the probability change in other two conditions between these two variables.

The significance of the entropy differences from different node selection measures was evaluated by t-test. The p-values between the entropy of the final learned Bayesian network structure from non-symmetrical entropy and other methods are all smaller than $10^{-10}$. This means that the entropy from non-symmetrical entropy is significantly smaller than others.

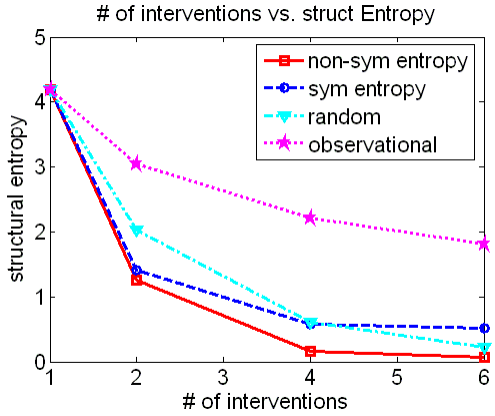


Fig. 2. Relationship between the number of interventions and the structure entropy of the learned Bayesian network from Cancer network. The non-sym entropy and the sym entropy refer to the node uncertainty measures with non-symmetrical entropy and symmetrical entropy defined in formulas (1) and (2), which are the same for other figures.

From Figure 2, we have a surprising observation. When the number of interventions is smaller than 6 in the Cancer network, the entropy of the learned structure with nodes selected from the symmetrical entropy is lower than that from random node selection. When the number of interventions is greater than or equal to 6, the entropy of the learned structure by node selection with symmetrical entropy is higher than that from random node selection. It means that, in the first several interventions, symmetrical entropy selects the nodes to reduce the structure uncertainty significantly when compared with random node selection. However, when the number of interventions is greater than 6, the leaf nodes (nodes $X_4$ and $X_5$ in Figure 1) are always selected by symmetrical entropy. The data with leaf nodes as interventional nodes can reduce the estimated probabilities of the edges from the nodes (as leaf nodes in the ground truth Bayesian networks) to other nodes. But, the data cannot provide information about the influence relationships from other nodes to the leaf nodes. The uncertainty of the leaf nodes calculated from symmetrical entropy can still be quite large. However, the random method may select other nodes for intervention, which could generate subsequent interventional data with more causal information about the edges from other nodes to leaf nodes and leaf nodes to other nodes. Such information will reduce the total structure entropy.

Figure 2 also shows that, with more interventions (which means more data), the entropy of the learned structure decreases with all the node selection criteria. The

entropy of the learned Bayesian network structure generally decreases more in the first few interventions. In the later stages, the entropy of the learned structure seems to converge to certain values. These results are similar across all the benchmark Bayesian networks tested.

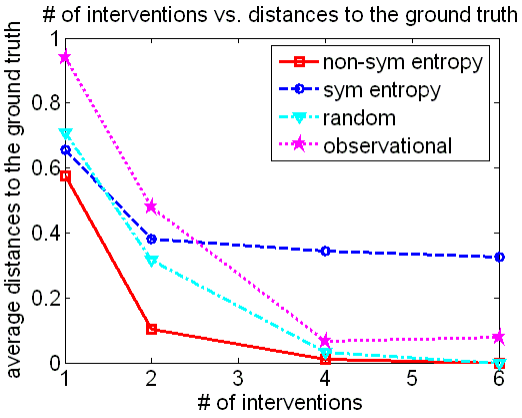


Fig. 3. Relationship between the number of interventions and the average hamming distance from the learned Bayesian network structure to the ground truth from Cancer network.

### 3.2 Number of interventions vs. Distance of the learned structure to the ground truth

In this experiment, we compared the learned structure with the ground truth Bayesian networks. The difference between the learned structure and the ground truth is measured with hamming distance. Figure 3 shows that node selection with non-symmetrical entropy leads to the smallest average hamming distance to the ground truth, as compared with other methods for node selection: symmetrical entropy, random node selection or observational data only. With 6 or more interventions with nodes selected by non-symmetrical entropy, the average distance is 0 and the variance is 0 with the Cancer network. The variances of the hamming distance from non-symmetrical entropy is the lowest, while the variances of the hamming distances from the symmetrical entropy and observational data are quite high (about 0.55 and 0.33 respectively). In addition, Figure 3 shows the changes of the average hamming distance with the number of interventions. With more interventional data, the average distance from the learned structure to the ground truth will be smaller.

From Figures 2 and 3, we can observe that, when the number of the interventions increases, the structure entropy converges to a certain low value with either node selection with non-symmetrical entropy or random node selection. The reason is that, when there are sufficient interventional data, either method can identify the true causal Bayesian network structure. We note that, however, when the number of interventions is small, non-symmetrical entropy could outperform all other methods for node selection in active learning. The difference in performance could be

significant in applications where data are scarce or only a small number of interventions are feasible.

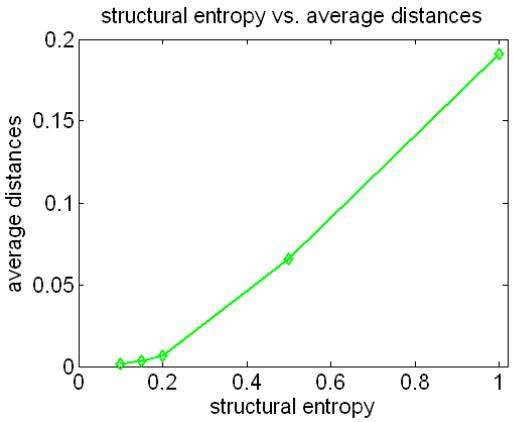


Fig. 4. Relationship between structure entropy of the learned Bayesian network and the hamming distance to the ground truth

### 3.3 Structure entropy vs. distance of the learned structure to the ground truth

In practice, we do not know the ground truth structure, and cannot use the hamming distance from the learned structure to the ground truth structure as the stop criteria to learn causal Bayesian networks. This experiment will examine the relationship between the structure entropy and the hamming distance from the learned structure to the ground truth Bayesian network structure. Figure 4 shows how the entropy of the learned structure approximates the average hamming distance from the learned structure to the ground truth. The relationship between the average entropy of the learned structure and the average distance from the learned structure to the ground truth is approximately linear, which means that the entropy of the learned structure is a good approximation of the distance of the learned structure to the ground truth Bayesian network and can be used as a stop criterion for the structure learning.

### 3.4 Structure entropy as stop criterion

In the next experiment, we tested the effect of the structure entropy as the stop criterion. Figure 5 shows that, with non-symmetrical entropy as the node selection criterion, the program can reach the required structure entropy with a smaller number of interventions. When the interventional node is selected with symmetrical entropy, a large number of interventions are needed. The results with observational data only do not show in Figure 5, as the program cannot reach the required structure entropy in the maximum steps allowed (50 steps) in that set of experiments.

A similar surprising observation appears in Figure 5: random node selection can reach the required structure entropy with smaller number of interventions than using symmetrical entropy for node selection. After investigating the intervention process

for node selection, we found that symmetrical entropy would select leaf nodes as the interventional node in many cases. Since the interventional data with the leaf nodes in the ground-truth Bayesian network intervened do not provide enough causal information to reduce the total structure entropy of the learned Bayesian network, some edge probabilities between the leaf node and other nodes may not converge to 0 or 1 with more data. In this situation, the leaf nodes can be selected for intervention again in node selection with symmetrical entropy, and the structure entropy of the learned structure cannot be reduced with more data. In the random node selection, variables other than the leaf nodes can be selected for intervention, which generate data with more causal information and can achieve the learned structure with smaller entropy. And with the non-symmetrical entropy, leaf nodes are only selected as the interventional nodes in a few rounds, because the probabilities from the leaf nodes to other nodes quickly converge to 0, and the non-symmetrical entropy will be near 0. This can explain why the non-symmetrical entropy is better than others for node selection in active learning.

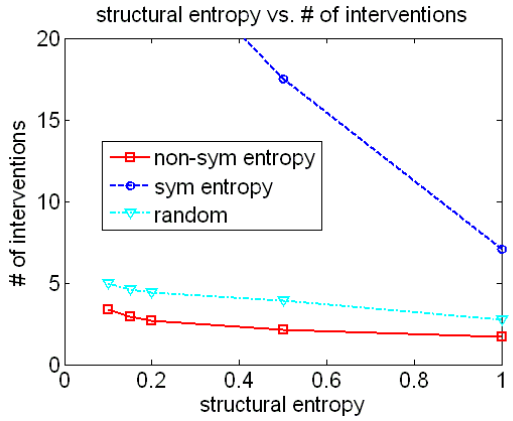


Fig. 5. Relationship of structure entropy and the number of interventions required from Cancer network.

### 3.5 Positive findings in subsequent interventions

In the final experiment, we considered the situation with resource constraints. In the previous experiments, the objective is to identify the whole causal structure with multiple interventions and we have examined different issues to reach this objective. In practice, there are usually resource constraints for interventions, and sometimes we can conduct only one interventional experiment. In this case, we hope to get a positive finding in this single interventional experiment which will show that there is really a causal relationship between the manipulated variable and one of the other variables.

The problem in this experiment is defined as follows: given the available data, domain knowledge and resource constraints, what is the likelihood to get a positive finding in a single interventional experiment? There is no guarantee to have a positive finding in a single experiment, but some strategies are available to increase the chance for a positive finding. In the experiment, we generated the observational data and

interventional data randomly first. Then we sampled the possible edges in the Bayesian network with probabilities 0.1, 0.2, 0.3 and 0.4 respectively as known edges (or domain knowledge). In this case, we assume we can only conduct one more interventional experiment. We estimated the edge probabilities with the available data, and chose as the interventional node the parent node of the edge with the highest probability. We repeated the experiments 1000 times in the different scenarios.

The results show that in above 98.5% cases, the edges with the highest probability from the available data and the known edges (as domain knowledge) are the true edges. It empirically shows that the edges with the highest probability are the best choice for a positive finding if we have resource constraints and only can conduct one more interventional experiment.

## 4 Discussion and Conclusion

In this work, we investigate active learning of Bayesian network structure when the interventional data is a data set at each active learning step, and propose using non-symmetrical information entropy to select nodes for intervention. Experiments show that non-symmetrical entropy can reach the required structure entropy with smaller number of interventions than symmetrical entropy and random node selection for intervention, and much better than merely estimating the structure with observational data in all three benchmark Bayesian networks tested. A possible reason for the better performance of the non-symmetrical entropy is that interventions are non-symmetrical in nature.

Experimental results also show that the learned structure entropy has an approximately linear relationship with the average hamming distance from the learned structure to the ground truth Bayesian network. This implies that the structure entropy is an effective measure for the goodness of the learned causal Bayesian network structure, and can be used as an effective stop criterion.

We have tested significance of the difference of the learned structure entropy from node selection based on the non-symmetrical entropy and other methods. The statistical test shows that the structure entropy from node selection with the non-symmetrical entropy is significantly smaller than that from other methods.

We have tested the possibility to have a positive finding when only a single intervention is possible due to resource constraints. In this case, experiment results show that the edges with the highest probabilities are usually the true edges given the available data and domain knowledge. It means that it is more likely to have a positive finding in next intervention by selecting the parent of the edge with the highest probability as the interventional node. In practice, if we can conduct one more intervention, our best choice is to choose the parent node of the edge with the high probability from the available data and domain knowledge. This will give us the best chance to have a real causal relationship discovery of the manipulated variable with one intervention.

A surprising observation in the experiments is that the random node selection for intervention can outperform the node selection with symmetrical entropy when the number of interventions is large. When the number of interventions is small, the

entropy of the learned Bayesian network structure with symmetrical entropy will be smaller than that from random node selection on average. When the number of interventions is large, the symmetrical entropy will often select leaf nodes for intervention, which cannot provide sufficient information to reduce the uncertainty of the edge probabilities. However, random node selection can select nodes other than leaf nodes for intervention, which can lead to the overall reduction in the uncertainty of the edge probabilities.

The closest related efforts to our work are those of Tong and Koller [14] and Eaton and Murphy [4]. Eaton and Murphy introduced uncertain intervention, but did not discuss active learning, even though they used a data set with both observational and interventional data. There are three main differences between our work and that of Tong and Koller [14]: 1) The interventional data collected at each active learning step is a data set, rather than a single instance; 2) As in Eaton and Murphy's work [4], we use the same exact method proposed by Koivisto [7] to estimate the edge probabilities, rather than MCMC, which can lead to better structure entropy estimation and node selection. The current exact method for edge probabilities can only be applied to cases with around 25 variables, while the MCMC method can be applied to cases with more variables. In our method, the edge probabilities can be estimated with MCMC method when the number of variables is large; and 3) We select the nodes for intervention based on the non-symmetrical entropy, not the expected posterior loss. We note that when the interventional data is a data set in each active learning step, it is not feasible to estimate the expected posterior loss due to the combinatorial problem of the possible data.

Our method is not designed to replace other related work, and does not apply to domains where repeated interventions are not possible, such as economics or social science. We have based our investigations on a set of different, complementary, or integrated situations with respect to the previous efforts [2, 4, 7, 14]: these efforts have also inspired some technical and presentation ideas reported in this paper. There are some general directions to extend our work, such as considering missing values or hidden variables in the causal Bayesian networks. In future, we will try to extend our results to more situations and apply the method to some real-life applications in different domains.

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